Comprehensive study of current controlled Memristor models

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***Abstract*—The mathematical postulation of memristor as fourth missing element in the fundamental electrical circuits,by Prof. Chua in 1971 was brought into reality with the physical device by HP lab in 2008.The nano scale memristor is getting considerable attention with its applications in diversified areas such as computational memorylogic circuits to neuromorphic applications . For the reliable implementation of memristor in complex circuits, various models of memristive systems are developed. This paper gives a brief review of existing models of current controlled memristor highlighting the features as well as the constraints of these models. A novel memristor model is proposed in the present paper which gives explicit equations for current-voltage relationship. The simulated results of the proposed model resemble the ideal non-linear hysterisis behaviour of memristor and are in good agreement with the I-V characteristics of fabricated Metal-Insulator-Metal (MIM) semiconductor devices with composition $pSi/TiN/TiO_2/Pt$.**

***Index Terms*—Current controlled memristor, memristive system, memristor model**

I. Introduction

MEMRISTOR was a novel concept postulated by Prof. Leon Chua in 1971 as the fourth missing circuit element in the list of three fundamental electric circuit elements such as Resistor, Capacitor and Inductor [1]. Basic mathematical relations for the fundamental circuit elements and memristor are as shown in Fig.1 and as summarized in Table 1.

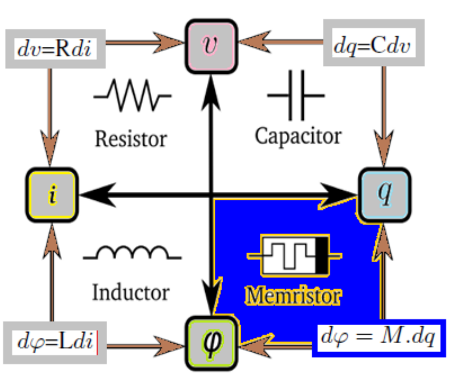


Fig. 1. The four fundamental electrical passive circuit elements:resistor ,capacitor,inductor and memristor

TABLE I
Mathamatical equation for fundamental circuit elements

Element	Derivative Function	Defination	Integral Function	Equation
R	dv/dt	$dv = Rdi$	$V = \int R\,di$	$V = IR$
L	$d\varphi/di$	$d\varphi = Ldi$	$\varphi = \int v\,dt$	$\int V\,dt = LI$
C	dq/dv	$dq = Cdv$	$q = \int I\,dt$	$\frac{1}{C}\int I\,dt = V$
M	$d\varphi/dq$	$d\varphi = Mdq$	$\varphi = \int V\,dt$	$\int V\,dt = M\int I\,dt$

Memristor was established as a non linear resistor wherein magnetic flux φ , is a function of charge q that has flown through it.The terms memristive systems and memristor are often used interchangeably for describeing memristive systems [2].A general Memristive system is described by set of two equations (1) and (2).

$$v = R(w, i).i \tag{1}$$

$$\frac{dw}{dt} = f(w, i) \tag{2}$$

Where v= memristor device voltage, i=memristor device current and $R(w, i)$ is the instantaneous resistance dependent on the internal state variable w of the device. By defination of current, $i = \frac{dq}{dt}$ and by Faraday's law of induction,$v = \frac{d\varphi}{dt}$ Equation (1) then can be modified to express relationship between magnetic flux linkage (φ) and electric charge (q). as

$$R(x, i) = \frac{v}{i} = \frac{\frac{d\varphi}{dt}}{\frac{dq}{dt}} = \frac{d\varphi}{dq} \tag{3}$$

Prof. Chua further defined two nonlinear functions $M(q)$ and $W(\varphi)$ and named it as memristance and memductance, respectively [2] Memristance $M(q)$ is the rate of change of magnetic flux with respect to charge. It is represented as $M(q) = (\frac{d(\varphi(q))}{dq})$ whereas Memductance $W(\varphi)$ is the rate of change of charge with respect to magnetic flux and is represented as $W(\varphi) = (\frac{d(q(\varphi))}{d(\varphi)})$ [6].Based on these relations memristors are classified as a current (or charge) controlled memristor and a voltage (or flux) controlled memristor and are represented by equations (4) and (5) respectively.

$$V = M(q).i \tag{4}$$

$$i = W(\varphi).v \tag{5}$$

where $q = \int_{-\infty}^{t} i\,dt$ and $\varphi = \int_{-\infty}^{t} v\,dt$. To design , analyze and simulate memristor based complex circuits and its implementation , many memristor models have been proposed by researchers [7] .These models are clasified as current controlled and voltage controlled models of memristor. This paper will discuss and compare the existing current controlled models of memristor and indroduce a novel model of memristor exhibiting ideal hysterisis behaviour.

II. HP Memristor model

A linear ion drift model for a memristive device was first suggested by HP research team.The realization of Memristor device by HP is in the form of thin TiO_2 insulator film sandwiched between two conducting metal plates of Platinum(Pt) to form Metal-Insulator- Metal (MIM) structure as shown in Fig 2. It was assumed that TiO_2 oxide layer has two regions ,TiO_2 (undoped) and TiO_{2-x} (doped) with oxygen vacancies(O^{+2}) . Undoped layer relates the insulating action with corresponding resistanceR_{off} (high resistance)and the doped layer TiO_{2-x} relates the conducting action with corresponding resistance Ron (low resistance).Width w corresponds to doped region and $(D - w)$ corresponds to undoped region.The resistive action of the physical memristor device can be understood from fig 3.On the application of possitive voltage ,repulsion of oxygen vacancies increases the size of doped region from w to D resulting in change in resistance from R_{off} to R_{on}(set condition).Similarly opposite polarity of applied voltage changes the resistance from R_{on} to R_{off}(Reset condition). [3]

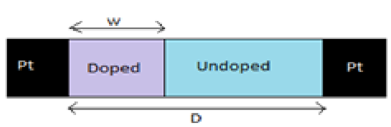


Fig. 2. Linear ion drift memristive device model by Hewlett-Packard is composed of doped and undoped regions

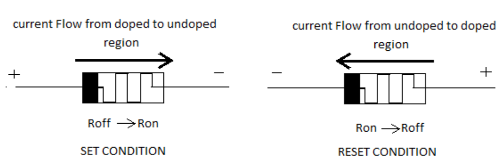


Fig. 3. Set-Reset condition of HP memristor

HP model was proposed on major assumptions such as liner ion drift in a uniform field,ohmic conductance and equal avarage ion mobility μ_v. A linear relationship between the drift-diffusion velocity and electric field is observed in HP model.With these assumptions, equation(1) and (2) can be represented by equation (6) and (7) respectively

$$V(t) = R_{on}.\frac{w(t)}{D} + R_{off}.\frac{(1 - w(t))}{D}.I(t) \quad (6)$$

$$\frac{dw}{dt} = (\frac{\mu_v.R_{on}}{D}).i(t) = k.i(t) \quad (7)$$

WhereR_{on} is resistance at condition $w(t) = D$ and R_{off} is resistance at $w(t) = 0$ with $(R_{off} >> R_{on})$; μ_v=drift mobility of oxygen vacancies ; $w(t)$ is the state variable and D is the thickness of the device.

In HP linear ion drift model to bind the state variable w within the limits of [0,D] and to add more nonlinear behaviour close to the bounds ,it was essential to introduce a window function.Window function helps to understand the effect of nonlinearity in the high electric field and effect of boundary condition.Several window functions were proposed by Strukov [4] ,Jogalekar [9], Prodomakis [5],Biolek [8] to overcome the boundary condition problem and absence nonlinear effect.

III. Nonlinear Models with window function

In the nanoscale devices small apllied voltage can yield enormous electric field.This can produce significant non linearities in ionic transport. To consider the nonlinear drift when w is close to 0 or D, equation (7) is multiplied by window function $f(x)$ [13].

$$\frac{dw}{dt} = (\frac{\mu_v.R_{on}}{D}).i(t).f(w) = k.i(t).f(w) \quad (8)$$

A. Strukov and Benderi window function

A simple Window function considering w as a state variable was suggested by Strukov [4] as

$$f(w) = \frac{w(1 - w)}{D^2} \quad (9)$$

which was later modified by Benderi and Wey [15] considering state variable x instead of w where, $x = w/D$ and was expressed as,

$$f(x) = x.(1 - x) \quad (10)$$

Equation (8) then becomes

$$\frac{dx}{dt} = (\frac{\mu_v.R_{on}}{D^2}).i(t).f(x) = k.i(t).x.(1 - x) \quad (11)$$

This modification nullifies the derivative of state variable x to reach zero at the boundaries, However,this modification does not suggest the nonlinearity at the boundaries.

B. Joglekar window function

To optimize the nonlinear behaviour of the function, Joglekar and Wolf [9] in 2009 developed a new window function with control parameter p as expressed in equation (12)

$$f(x) = 1 - (2x - 1)^{2p} \quad (12)$$

Where p is positive coefficient .The characteristics of window function becomes near to rectangular function as p increases. Optimization is achieved by sweeping parameter p from 0 to 10. Optimum value of p for Joglekar window is 9.88 [14] .However, this window function observed terminal state problem, which restricts the memristor to come back from terminal state even when a reversed bias input is applied.This contradicts the fundamental non-volatile property of memristor.

C. Prodomakis window function

Prodomakis suggested a new control parameter j to make the window function scalable which was not possible with the previous window functions [5] . It was helpful to overcome the restriction on the maximum value of window function to be unity with control parameter ‘j’parameter . Equation (13) represents Prodomakis window function

$$f(x) = j(1 - [(\frac{w}{D} - 0.5)^2 + 0.75]^p) \tag{13}$$

where, j decides the highest value of window function. The optimized value of j and p are -47.37 and 99.94 respectively by sweeping p from 0 to 100 [14]. With Prodomakis model it is observed that the effect of nonlinearities at the boundaries is optimised by value of ‘p’ whereas the scaling of magnitude of window function is achieved by parameter ‘j’.

D. Biolek window function

For all above window functions terminal state problem was prominently observed . Further, these window functions were considering only the changes in state variable but not the charge passing through the physical memristor device. Considering these constraints Biolek suggested a novel current dependent window function as represented by equation(14).

$$f(x) = 1 - [x - stp(-i)]^{2p} \tag{14}$$

where $stp(i)$ is a step function of current .This function allows the change in the value of window function at terminal state from minimum to maximum value after the application of reverse biased input.This resolves the terminal state problem. However, with Biolek’s window function there is no continuity at the boundaries and also the ideal symmetrical hysteresis behaviour is not observed. The major constraint observed in all the models discussed was their inability to capture the highly nonlinear behaviour [13] observed in the physical memristor devices.This constraint was addressed in [11] on the basis of simmons tunnel barrier effect. [16]

IV. SIMMONS TUNNEL BARRIER MODEL

Simmons current controlled memristor model represents nonlinear ion drift effects and asymmetric switching behaviour with the help of tunneling effect [11]. This model is explained on the basis of drift and diffusion of oxygen vacancies.Equation (15) represents the state equation for simmons tunnel barrier model.

$$\frac{dx}{dt} = \begin{cases} c_{off}.\sinh(\frac{i}{i_{off}}).e^{[-e^{(\frac{x-a_{off}}{w_c} - \frac{|i|}{b})} - \frac{x}{w_c}]} & \text{if } i > 0 \\ c_{on}.\sinh(\frac{i}{i_{on}}).e^{[-e^{(\frac{x-a_{on}}{w_c} - \frac{|i|}{b})} - \frac{x}{w_c}]} & \text{if } i < 0 \end{cases} \tag{15}$$

Where $\frac{dx}{dt}$ is drift velocity of oxygen vacancies ;$C_{on} >> C_{off}$ represents the magnitude change in ’x’ ;a_{on} and a_{off} are boundaries of Simmons tunnel barrier width ’x’; $i_{on}$and i_{off} are positive and negative threshold current respectively.This model discussed asymmetric behavior of practical memristor in respect of variation in different drift and diffusion of oxygen vacancies after the application of positive and negative bias voltages .It was shown for the first time that drift and diffusion are in opposite direction for positive bias voltage while for negative bias voltage it is in same direction. [11] Simmons tunnel barrier model is claimed to be the most accurate physical model of memristor. However,it is device specific and not applicable for all types of memristor. Because of the complexity, ambiguous nature of voltage and current relationship and computational inefficiency of the Simmons model , there is a need of developing new model with simpler mathematical functions keeping the accuracy of simmons model intact. [12]

V. TEAM MODEL

With the simplified property of mathematics Kvatinsky proposed a new model based on current threshold. This model is modification of Simmons model with separate window function for ON and OFF state. The state equation of THreshold Adaptive Memristor model [12]is given as in equation(16).

$$\frac{dx}{dt} = \begin{cases} k_{off}.((\frac{i}{i_{off}}) - 1)^{\alpha_{off}}.f_{off}(x), & 0 < i_{off} < i \\ 0 \qquad , & i_{on} < i < i_{off} \\ k_{on}.((\frac{i}{i_{on}}) - 1)^{\alpha_{on}}.f_{on}(x), & \text{if } i < i_{on} < 0 \end{cases} \tag{16}$$

where a_{on} and a_{off} represent the boundaries of ‘x’; $f_{on}(x)$ and $f_{off}(x)$ represent window functions for ON and OFF state respectively as given by equation (17) and (18).

$$f_{on}(x) = e^{-e^{(\frac{x-a_{on}}{w_c})}} \tag{17}$$

$$f_{off}(x) = e^{-e^{(\frac{x-a_{off}}{w_c})}} \tag{18}$$

The explicit behaviour of voltage and current for TEAM model is represented by equation (19)and (20)

1.Linear fit :

$$v(t) = [R_{on} + \frac{(R_{off} - R_{on})}{(x_{off} - x_{on})}.(x - x_{on})].i(t) \tag{19}$$

2.Exponential fit:

$$v(t) = R_{on}e^{(\frac{\lambda(x-x_{on})}{x_{off}-x_{on}})}.i(t) \tag{20}$$

Where, $\lambda = \ln \frac{R_{off}}{R_{on}}$. TEAM model thus describes the linear as well as the desired exponential behaviour of current and voltage .

VI. POLYNOMIAL MODEL

A model based on Prof.Chua’s definition of memristor and expressed with simple polynomial representation of state equation was proposed as polynomial model (PM) in 2013 [14].The parameters in PM are obtained by Taylor series expansion of reference model with current compliance for fixed state of memristor.The state equation of model reveals its dependency on polynomial parameters including current i as observed in equation (21)

$$\frac{dx}{dt} = \sum_{n=0}^{N} a_n.i^n + \sum_{m=0}^{M} b_m.w^m.i + \sum_{p=1}^{P} (c_p.i + d_p)^p (e_p.w + f_p)^p \tag{21}$$

TABLE II
COMPARISON OF CURRENT CONTROLLED MEMRISTOR MODELS

Sr.No	Model Name	State Variable	Control Parameter	Reference model used	Features	Constraints
1.	**Linear model** HP's linear ion drift model	w	w	HP	linear relation of drift velocity and electric field	Non linearity is not considered
2.	**Nonlinear models**: I. Strukov	w	w	HP	Introduction of window function to acieve nonlinearity .State variable w bound within [0,D]	Does not satisfy boundary condition at $w = D$
	II. Benderi and wey	x	x	HP	Satisfy boundary condition at $w = 0$ and $w = D$	Expected nonlinear effect is not achieved
	III.Joglekar	$x = \frac{w}{D}$	p	HP	Introduction of control parameter p to optimise nonlinear effects ,Binds x in $[0, 1]$ with increased value of p	Terminal state problem restricting the nonvolatility of memristor
	IV.Prodomakis	$x = \frac{w}{D}$	j, p	HP	Scalable window function with parameter j which overcomes the restriction on maximum value of unity	Terminal state problem,window function considering changes only in state variables but not in charges
	V.Biolek	$x = \frac{w}{D}$	$stp(-i)$	HP	Introduction of $stp(-i)$ to consider charge variation,terminal state problem resolved	No continuity of function at boundary ,Non linearity not clearly observed
3.	**Simmons tunnel barrier Model**	Tunnel barrier width(x)	i, x	Tunnel barrier model	Use of tunneling effect for asymetric switching behaviour of memristor	Device specific,no explicit I-V relationship ,Complex, computationally inefficient
4.	**TEAM Model**	Tunnel barrier width(x)	i, x	Tunnel barrier model	Introduction of current threshold concept, Explicit V-I relationship for linear and exponential fit	State equation based on continuous polynomial functions
5.	**Polynomial Model**	w	i, w	HP	Simple polynomial state equation,results matching with HP and TEAM model	No physical implementation is achieved
6.	**Proposed model**	x	$stp(-\frac{V}{R_{on}}), m$	HP	Explicit I-V relation,results in agreement with fabricated devices	Device specific

PM model highlights the results that match with the existing TEAM and HP model. The comparitive study of various current controlled memristor models suggests that it is essential to develope a new model having simple mathamatical equations with explicit current and voltage relationship and keeping the nonlinear behaviour of memristor intact.Such model will lead to reliable implementation of memristor in complex circuits.Table II gives the comprehenssive summary of various models of current controlled memristor highlighting the features as well as the constraints

VII. Proposed model

In the developement of nonlinear current controlled models of memristor, various window functions are observed are implemented to overcome terminal state problem and to achieve the expected nonlinearity.Window functions are considered as a major of preciseness for storing the amount of electric charge without loosing the memory effect.Joglekar [9] and prodomakis [5] models introduced the new concept of control parameter and scalable window function.Biolek's model considered the changes in charge variable which was missing in previous models of memristor.It was indroduced with the control parameter $stp(-i)$ in the window function [8]. However,It is neccessary to consider the parameter which is responsible for switching of the device in to on state i. e.R_{on}, to exhibit the memresistive effect. Considering this fact we propose a new window function

$$f(x) = 1 - [x - stp(-\frac{V}{R_{on}})]^{2p} \tag{22}$$

where R_{on} is low resistance of doped region responsible for the switching action.Substituting the new value of window function in equation(11) and solving the differential equation,we get explicit relationship between voltage and current as expressed in equation(23)

$$I = x^m.V + (1 - x)^m.V \tag{23}$$

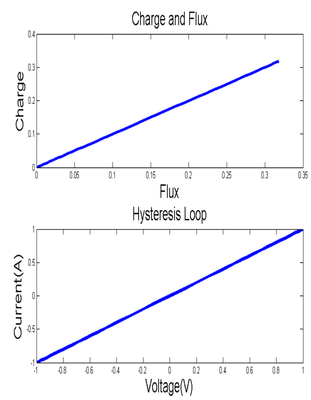
where the 'm' is control parameter for nonlinear behaviour of memristor. The proposed model offers an explicit equation of voltage and current which was not clearly observed in the previous nonlinear memristor models,where HP model was used as reference model.

In table III shows the MATLAB simulation results of proposed model. From the simulation results,nonlinearity is found to be a function of control parameter 'm'.With increase in the value of 'm' the non linearity improoves with a corrosponding decrease in current.The ideal hysterisis behaviour observed in the simulated results is in aggrement with the fundamental memristor property [1].

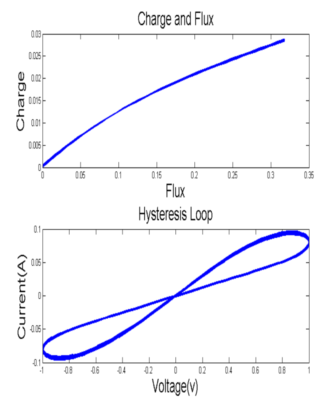
The simulation results in table IV resemble physical behaviour of resistive switching observed in a fabricated nanoscale memristor devices with composition $pSi/TiN/TiO_2/Pt$ in which the thin TiO_2 films(12nm) were deposited with advanced technique of Plasma Enhanced Atomic Layer Deposition(PEALD)

TABLE III

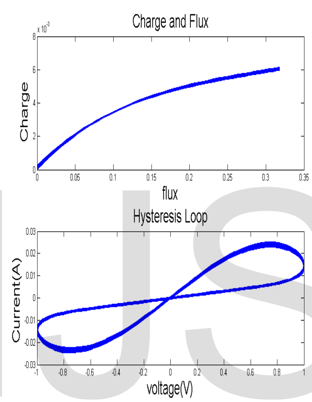
EXPLICIT CURRENT VOLTAGE RELATIONSHIP AND CHARGE FLUX CHARACTERISTICS OF PROPOSED MODEL SHOWS THE NONLINEARITY CONTROLLED BY PARAMETER 'm'' (SWEEPING VALUES OF m FROM 1 TO 15)



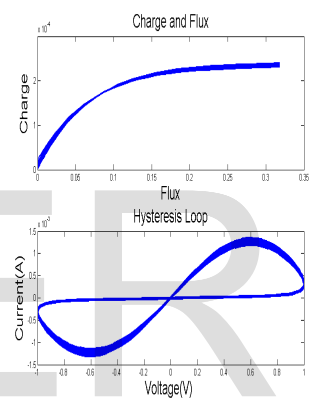
(a) $m = 1$



(c) $m = 5$



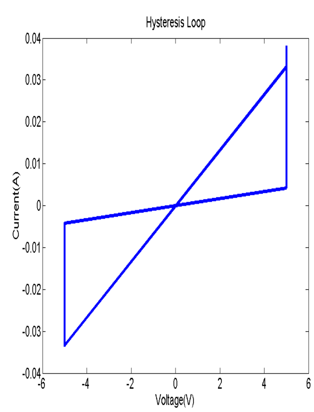
(d) $m = 8$



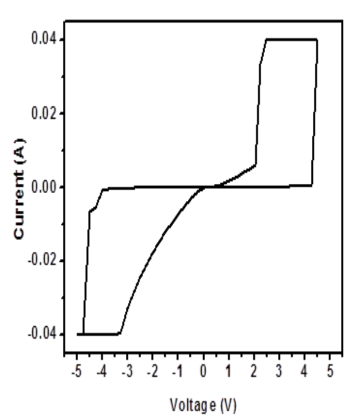
(f) $m = 15$

TABLE IV

CURRENT VOLTAGE RELATIONSHIP OF PROPOSED MODEL SHOWS RESEMBLENCE WITH ACTUAL PHYSICAL MEMRISTOR WITH COMPOSITION OF $pSi/TiN/TiO_2/Pt$



(a)



(b)

VIII. Conclusion

A nonlinear memristor model that controls the nonlinearity and offers explicit relation between voltage and current is required to exhibit the ideal hysterisis behaviour.In this paper we have discussed and compared various available current controlled memristor models with reference to basic assumptions,mathamatical equations, special features and model constarints.A different approach of memristor model is proposed to get explicit current and voltage relationship which exhibits the the physical switching behaviour of a fabricated nanoscale memristor devices with composition $pSi/TiN/TiO_2/Pt$.The proposed model will be useful for future reliable implementation of memristor in complex circuits.

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